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ABSTRACT

In this paper different methods proposed for determination of k_s are discussed, compared and evaluated for their suitability and accuracy. The geotechnical parameters of a site on Tabriz Marl were selected as the base data and settlement analysis results with different methods were compared with that of obtained from analyses with advanced soil models using Safe and Plaxis soft wares. It was disclosed that for Tabriz Marl, soft soil model is the best governing model and Vesic relation among the methods of determination of k_s leads to a negligible error in comparison to the soft soil model. Also, in order to achieve more accurate results from these methods, it is proposed to use mean elasticity modulus which takes into account the effect of geometric and mechanical properties of sub-layers.

INTRODUCTION

Because of the complexity of soil behavior, subgrade in soil-foundation interaction problems is replaced by a much simpler system called subgrade model. One of the most common and simple models in this context is Winkler hypothesis (1867) which is well-known among the majority of designers. Winkler idealization represents the soil medium as a system of identical but mutually independent, closely spaced, discrete and linearly elastic springs and ratio between contact pressure, P, and settlement, y, produced by it at that point, is given by the coefficient of subgrade reaction, k_s (Dutta and Roy, 2002).

Evaluation of the numerical values of k_s is one of the most complex and sophisticated problems in geotechnical engineering. In the other hand, this factor leads to inaccuracy in the results of Winkler model and this aspect of the problem is scrutinized in this paper by a case study. The k_s is not a fundamental soil property and it is a problem-specific observed result and in addition to depending on elastic characteristics of subgrade, it also relates to the geometry of the foundation and loading scheme (Terzaghi, 1955). Particularly between the 1950s and 1980s, this concept has been scrutinized and numerous relations have been proposed by investigators. (Daloglu and Vallabhan, 2000).

Nevertheless, there is not enough information in technical literatures about the computational validity and accuracy of comprehensive application of these relations in engineering practice. Hence, in this paper different methods, proposed for determination of k_s, are compared and evaluated for their suitability and accuracy. The geotechnical parameters of a site on Tabriz Marl were selected as the base data and settlement analysis results with these methods are compared with that of obtained from analysis with advanced soil models.

Among the numerous relations that have been proposed, the equation obtained from the theory of elasticity, Biot relation and Vesic relation are more appropriate for evaluation of k_s in this study (Akbarzad, 2006, Bowles, 1998). These relations are presented in Table 1. Hence, in this paper accuracy and precision of these relations in predicting settlement and contact pressure are evaluated in detail.

Table 1. Common relations of determination of k_s

No.	Source	Suggested relation
1	Biot (1937)	$k_s = \frac{0.95 E_s}{B(1-\nu_s^2)}\left[\frac{B^4 E_s}{(1-\nu_s^2)EI}\right]^{0.108}$
2	Vesic (1961)	$k_s = \frac{0.65\ E_s}{B\ (1-\nu_s^2)}\sqrt[12]{\frac{E_s B^4}{EI}}$
3	Theory of elasticity	$k_s = \frac{E_s}{B'(1-\nu_s^2)mI_s I_F}$

In Table 1, E_s = modulus of elasticity, υ_s = Poisson’s ratio, B = width of foundation, EI = flexural rigidity of foundation, B' =

least lateral dimension of footing, I_S and I_F = influence factors which depends on the shape of footing and m takes 1, 2 and 4 for edges, sides and center of the foundation, respectively. Equation 3 is obtained from the relation of settlement of rectangular plates resting on elastic half space (Biot, 1937, Vesic, 1961, Bowles, 1998).

For analyzing based on Winkler model and advanced soil models, Safe v. 8.06 and Plaxis v. 7.2 soft wares are used, respectively. In plaxis soft ware, advanced soil models consisting soft soil, creep soft soil and Mohr-Coulomb model may be applied.

GEOTECHNICAL PROPERTIES OF GROUND AND SOIL PARAMETERS EMPLOYED FOR MODELIN

The examined project includes a 22-story residential building that will be constructed on a 44×20 m rectangular mat footing. The geometry and loading are symmetric. Site is placed in southeast of Tabriz city in Iran. The mat will be founded 6 m below the original ground level. A detailed site investigation was carried out to provide the required engineering information and description of subsurface soil. These are summarized in Table 2. Referring to the consolidation test results, it was observed that the soil is over consolidated with pre-consolidation pressure values 900 kPa and 950 kPa for yellow marl and gray marl, respectively.

DETECTING THE SUITABLE MODEL

Suitable soil model for yellow marl and gray marl were examined separately and are shown in Fig. 1. This was carried out by comparing the consolidation test results of those soils and the stress-settlement curve obtained from modeling of the consolidation test in Plaxis with different soil models. For all the models, identical average values of density (γ_{wet}, γ_{dry}) and failure parameters (C', φ') were defined. The angle of dilation was assumed to be zero, however.

For Mohr-Coulomb model variations of modulus of elasticity E_s with effective stress level are based on the data obtained from consolidation test. In soft soil creep and soft soil models, relevant values of required parameters are also assigned using the results of consolidation tests (Manual of Plaxis). Loading steps are applied same as the laboratory tests. The results of these analyses together with the average of the measured laboratory data are plotted in Fig. 1 for comparison.

Table 2. Soil properties and description

No. of layers	Depth (m)	Soil description	Moisture content (%)	γ_d (kN/m^3)	C' (kN/m^2)	φ' (°)	PI (%)	LL (%)
1	0-11	Weakly cemented silty sand and gravel, water table at 8.0 m.b.g.l.	8	18	0	35	-	-
2	11-14	Weathered yellow marl	64	9	55	21	77	45
3	14-17	Yellow marl	55	12	76	20	72	41
4	17-19.7	Yellow- Greenish marl	67	10	60	20	75	47
5	19.7-23	Fissured gray marl	67	11	54	20	75	45
6	23-25	Dark gray marl	72	9	79	20	72	40

Based on Fig. 1, for the both marl soils, the soft soil model shows better coincidence to the mechanical behavior in comparison to other advanced soil models. Also, it corresponds to empirical observations, because the examined soil mass, in common with soft soils, exhibits comparatively high degree of compressibility (Sadrekarimi and kia, 2005). Therefore, the soft soil model is used as a comparison criterion in the subsequent analyses.

SETTLEMENT ANALYSIS USING SOFT SOIL MODEL

In order to consider effect of layering and mechanical properties of subsoil on ground settlement, the geometry modeled in Plaxis soft ware was extended down to the influence depth of the foundation which is given to be five times of foundation width 5B (Bowles, 1998). In order to consider stress-history caused by excavating, mat was modeled 6 m below the original ground level. Layering and soil properties were defined referring to Table 2. Since properties of soil down to the depth of 25 m are available, texture and engineering properties of ground down to the influence depth of super structure, regarding the local information on Tabriz subsoil zonation, is assumed to be the same as the layer No. 6. In soft soil model, variation of modulus of elasticity with effective stress considered linear (Manual of Plaxis). The soil mass from 23 m to 106 m deep was divided into several layers in a manner that the error due to assuming linear variation of E_s with stress level became negligible. For sandy layer Mohr-Coulomb model was employed and E_s value beneath the foundation level was estimated as 116040 kPa using the results of SPT tests. Finally, the deformed mesh is illustrated in Fig. 2.

EVALUATING THE COEFFICIENT OF SUBGRADE REACTION

Main problem with the accuracy of k_s relations is related to evaluation of E_s. This is due to the fact that the modulus of elasticity is the only factor by which the effect of subsurface soil properties on the value of k_s can be examined. Hence equivalent modulus of elasticity which involves the mechanical properties of the layers within the influence depth should be assigned.

It is obvious that the effect of external load decreases with depth (Bowles, 1998). Hence, moduli of elasticity of upper layers are more effective on deformation settlement than the lower layers. This issue is named depth factor, I_{Di}, in this paper. Evaluation of the equivalent modulus of elasticity consists of two steps: assigning the effect of geometric properties of layers and characterizing the value of depth factor. For the first one, thickness of each layer is selected and depth factor is defined as a ratio of settlement at mid-point of thickness of each layer to total settlement of the geometry modeled in Plaxis software (Fig. 2) and equivalent modulus of elasticity, E_{se}, is given by

$$E_{se} = \frac{\sum_{i=1}^{10} E_{si} I_{Di} H_i}{\sum_{i=1}^{10} I_{Di} H_i} \tag{4}$$

in which E_{si} = modulus of elasticity at mid-point of thickness of each layer and H_i = thickness of each layer. Substituting relevant values E_{se} is obtained equal to 21021 kPa. Whereas if one disregards layering, E_s along soil-foundation interface equals to 116040 kPa. It is evident that the significant difference between these values will lead to a remarkable error in predicted settlement. This indicates the importance of layering in determination of k_s.

a)

b)

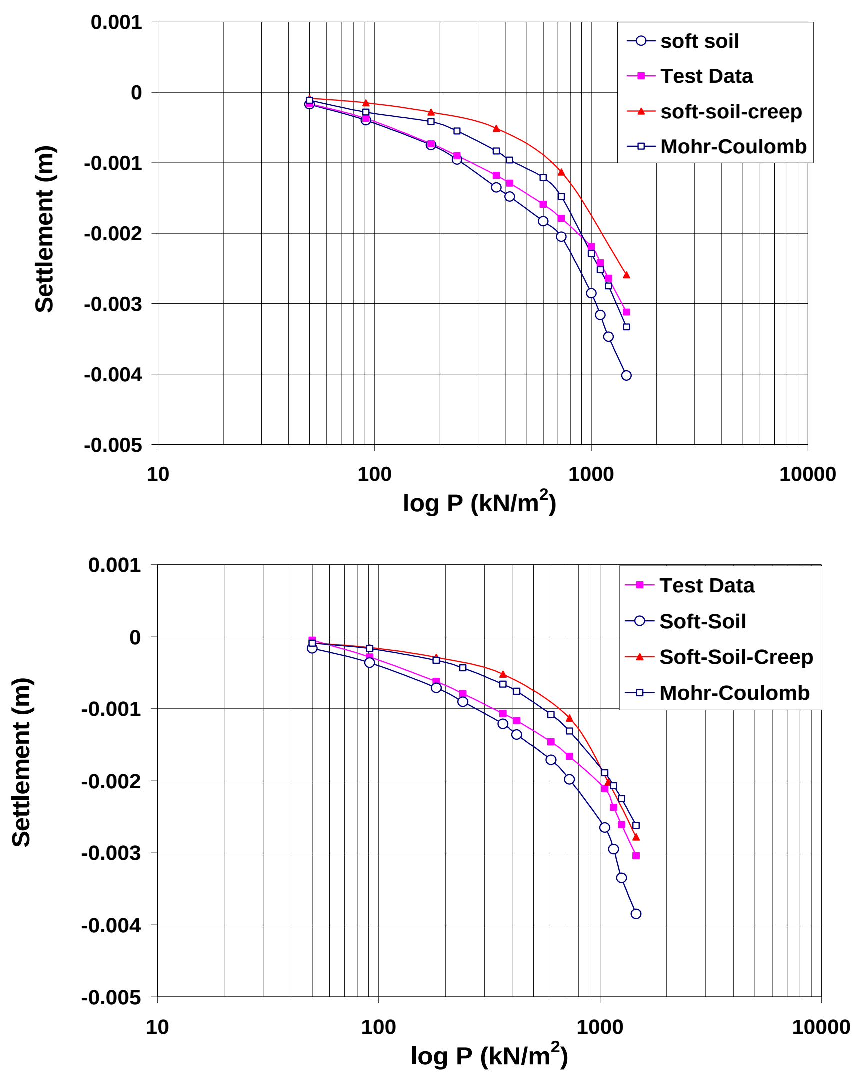

Fig. 1. Comparison between stress-settlement curves obtained from advanced soil models and results of consolidation test on a) Yellow marl, b) Gray marl

Substituting relevant values for other parameters in equations 1, 2 and 3 k_s were computed as 1419 kN/m^3, 980 kN/m^3 and 1500 kN/m^3, respectively. It should be noticed that the relation obtained from the theory of elasticity (Equation 3) gives various quantities for edges and center of the foundation. Therefore for estimating the average coefficient of subgrade reaction the suggested method by Bowles (1998) is used.

Settlement analyses were executed for each set of data given above with a soft ware called SAFE. The foundation is modeled as a rectangular plate and loading was defined the same as the other ones which were used in Plaxis. Because of using of plane-strain analysis in Plaxis, the foundation is considered as a 20 m long strip with unit width. In order to reduce the inaccuracy, settlement and contact pressure beneath the central strip of the foundation, obtained from Winkler and the soft soil models, were compared.

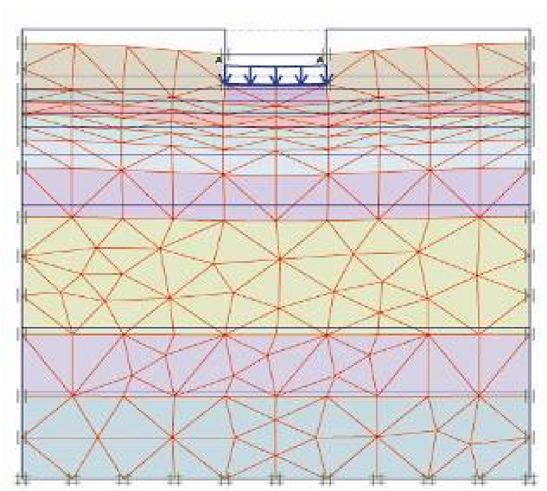

Fig. 2. The Geometry modeled in plaxis and deformed mesh

COMPARISON OF THE RESULTS

Settlement and contact pressure diagrams obtained from the soft soil and Winkler models are presented in Figs. 3, 4 and 5. In the given soil mass, differences between settlement and contact pressure obtained from the theory of elasticity and Biot relation are negligible. Since the soft soil model is intended as a criterion of accuracy of the determination relations of k_s, it can be concluded that the Vesic relation predicts settlement with acceptable accuracy for using in Winkler method. This relation gives the maximum settlement 8 % greater than that of the soft soil model. However, relation obtained from theory of elasticity and Biot relation estimate the settlement 30% and 34% less than that of the soft soil model, respectively. Vesic relation can be proposed as one of the main alternatives in predicting the behavior of yellow and gray Marl. Nevertheless, remedial measures are necessary to control settlement of the foundation (Fig. 4), but the main purpose of this paper is comparison of the results, so this issue is disregarded.

Interpreting the results of contact pressure is a little more complex. In Winkler model, elasticity, Biot and Vesic relations lead to approximately equal values of contact pressures. But the values obtained from Winkler model have great difference with that of the soft soil model. Winkler approach gives the maximum contact pressure 35 % greater than the soft soil model does. The difference is derived from ignoring the lateral pressure of soil in Winkler model. Because lateral pressures of soil elements on the soil-foundation interface reduce the vertical pressure whereas this feature is modeled in the soft soil model (Manuel of Plaxis). But, in reality, lateral pressures by surrounding the soil around the foundation decrease the vertical pressure. Consequently it is expected that Winkler model leads to larger settlement as well. While mostly it gives smaller values. This may be attributed to the fact that the methods of determination of k_s used in common practice, estimate it in a way that magnitude of settlement is not influenced (Akbarzad, 2006).

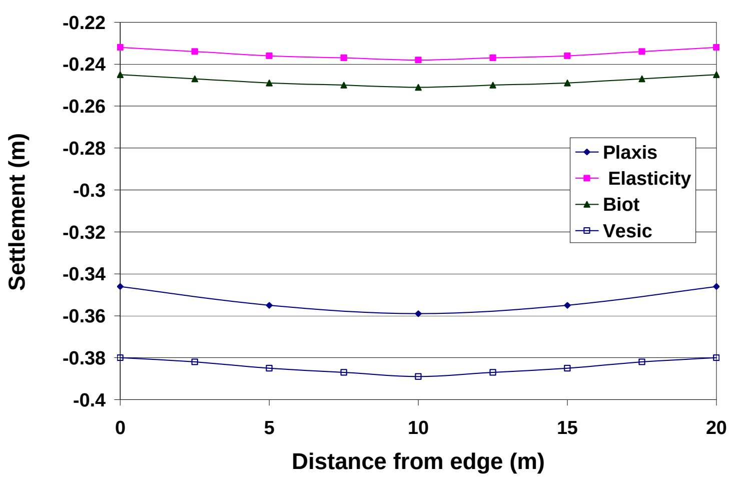

Fig. 3. Settlement diagram

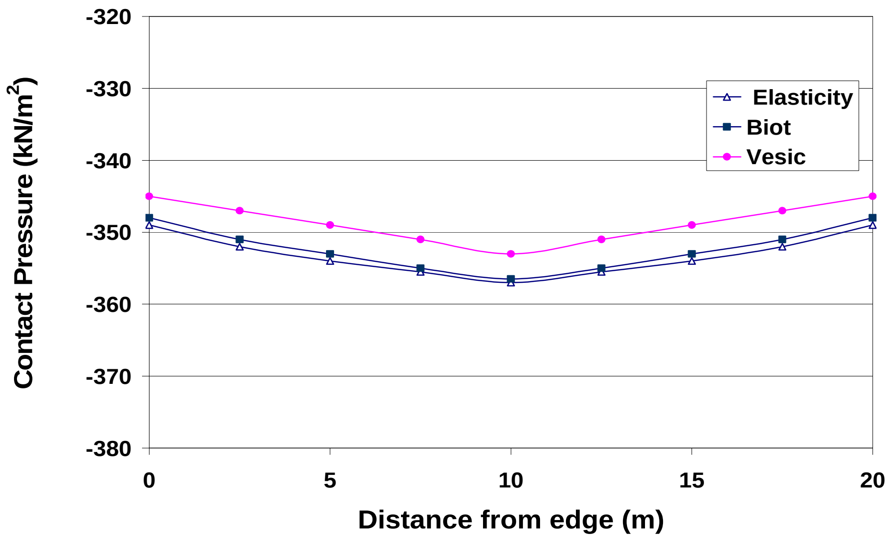

Fig. 4. Contact pressure diagram

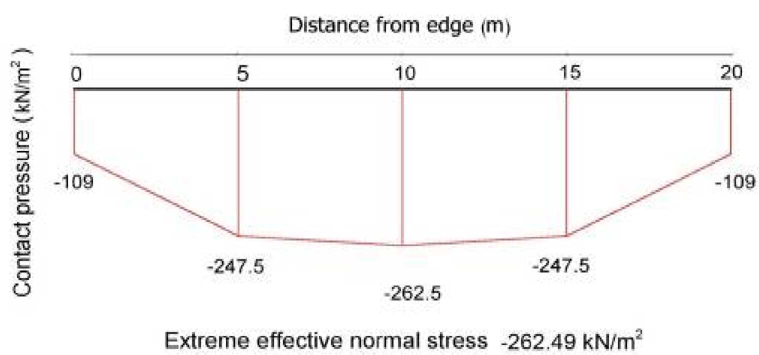

Fig. 5. Contact pressure diagram obtained from the soft soil model

Nevertheless, in this article there is not any plate-load test result, but evidently in this test only mechanical properties of the layers placed within the influence depth of the loading plate, which is too small in comparison with the actual size of a foundation, affects k_s value. It can be concluded that if the rate of the variation of E_s with respect to depth is considerable, results of plate-load test cannot be reliable.

CONCLUSIONS

1- The coefficient of subgrade reaction is a concept that is valid only at soil-foundation interface, but in this article, in order to increase the accuracy of the results, the effect of layering and mechanical properties of the subsurface soil on k_s are dealt with.

2- Among the methods for determination of k_s value, Vesic relation leads to acceptable accuracy in evaluating settlement in comparison to the soft soil model. Accordingly, this relation is suggested as a governing relation for estimating k_s for the given soil mass.

3- Winkler relation gives contact pressure greater than actual values and it is derived from disregarding the effect of lateral pressures of soil mass.

4- In common practice, in order to minimize inaccuracy of k_s relations, two items should be considered. At first one should have vast study and awareness on the basic theories of these relations; and secondly, in addition to geometric properties of layers, variation of the mechanical properties with depth is also considered in evaluation of the equivalent modulus of elasticity.

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